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# PERFORMANCE STUDY OF A PIER BY 'FBD' AND 'DDBM' METHODS & PARAMETRIC STUDY OF CURVED BOX GIRDER

**Rajesh Kumar[1], Ajay Kumar Singh[2],**

[1] PG Student, [2]Head of Department
Roorkee Institute of Technology
Roorkee

## ABSTRACT

A metro system is a railway transport system in associated populated area with a high capability, frequency and the grade separation from different traffic. Metro System is used in cities, agglomerations, and metropolitan areas to move large no. of people. Associated elevated metro system is more referred type of metro system due to ease of construction and also it makes urban areas more accessible without any construction difficulty. An elevated metro system has two major elements pier and box girder. The resent study focuses on 2 major elements, pier and box girder, of an elevated metro structural system.

The performance assessment of selected designed pier showed that, the Force Based Design Method may not always guarantee the performance parameter required and in the resent case the pier achieved the target requirement. In case of Direct Displacement Based Design Method, selected pier achieved the behavior factors more than targeted Values. These conclusions may be considered only for the selected pier.

The parametric study on behavior of box girder bridges showed that, as curvature decreases, responses admire longitudinal stresses at the top and bottom, shear, torsion, moment and deflection will decreases for 3 types of box girder bridges and it shows not much variation for fundamental frequency of three types of box girder bridges due to the constant span length. It is observed that as the span length increases, longitudinal stresses at the top and bottom, shear, torsion, moment and deflection increases for three types of box girder bridges. As the span length increases, fundamental frequency decreases for three types of box girder bridges. Also, it is noted that as the span length to the radius of curvature ratio increases responses parameter longitudinal stresses at the top and bottom, shear, torsion, moment and deflection are increases for three types of box girder bridges. As the span length to the radius of curvature magnitude relation will increases fundamental frequency decreases for 3 types of box girder bridges.

## INTRODUCTION

A metro system is an electric passenger railway transport system in an urban area with a high capability, frequency and the grade separation from alternative traffic. Metro System is used in cities, agglomerations, and metropolitan areas to transport large numbers of individuals at high frequency. The grade separation allows the metro to move freely, with fewer interruptions and at higher overall speeds. Metro systems are typically located in underground tunnels, elevated viaducts above street level or grade separated at ground level. An elevated metro structural system is more referred because of simple construction and also it makes urban areas more accessible without any construction difficulty. An elevated metro structural system has the advantage that it is more economic than an underground metro system and the construction time is much shorter.

An elevated metro system has 2 major elements pier and box girder. A typical elevated metro bridge model is shown in Figure (a). Viaduct or box girder of a metro bridge requires pier to support each span of the bridge and station structures. piers are constructed in various cross-sectional shapes like cylindrical, elliptical, square, rectangular and other forms. The piers considered for the resent study are in rectangular cross section and it is located under station structure. A typical pier considered for the resent study is shown in Figure (b). Box girders are used extensively in the construction of an elevated metro rail bridge and the use of horizontally curved in plan box girder bridges in modern metro rail systems is quite suitable in resisting torsional and warping effects induced by curvatures. The torsional and warping rigidity of box girder is due to the closed section of box girder. The box section also possesses high bending stiffness and there is an efficient use of the entire cross section. Box girder cross sections may take the shape of single cell, multi spine or multi cell as shown in Figure below.

Typical Elevated Metro Bridge

(a) Typical pier

## SIGNIFICANCE OF THE STUDY

A force based seismic design approach is conventionally used to design the railway line bridge

pier. Throughout a seismic loading, the behavior of elevated bridges depends totally on the ductility and therefore the displacement capacity of the pier. It is important to check the ductility of such single piers. Force based method don't explicitly check the displacement capacity at the design stage. The codes are now moving towards a performance-based (displacement-based) design approach, which consider the design as pier the target performances at the design stage.
The behavior of a box girder curved in arranges is significantly different from a straight bridge and its dependent on many parameters. A limited number of studies are conducted on this aspect.

**OBJECTIVE:**

- To study the performance of a pier designed by Force Based Design Method (FBD) and Direct Displacement Based Design (DDBD) Methodology.
- To study the parametric behavior of a Curved Box Girder Bridges.

**SCOPE**

- The resent study is restricted to those practical cases that come across in an elevated metro project.
- With regard to the geometry of the pier considered, the current study is limited to O Rectangular pier cross section O Single pier structural system O Reinforced concrete pier
- Parametric Study on Box Girder is limited to, O Linear static and dynamic analysis and Nonlinear analysis isn't considered O Rectangular box section with flanges O Reinforced concrete box girder section and not applicable to re-stressed bridges. O Single Cell and Multi Cell Box Girder and not applicable to Multi Spine box girder.O 0% gradient of the superstructure and super elevation is not considered within the modeling

**PERFORMANCE STUDY OF A PIER DESIGNED BY FBD AND DDBD**

Performance study of the typical pier designed by a Force Based Design (FBD) Method and Direct Displacement Based Design (DDBD) Method is described in this paper. The pier is designed based on FBD and DDBD Method. performance assessment is carried out for the designed pier and the results are discussed briefly.

**DESIGN OF PIER**

Piers shall be designed to withstand the load and forces transferred from the superstructure and the load and forces on the pier itself, apart from the effect of its self-weight. In general, pier may be solid, hollow or framed structures. Conventionally the pier of a metro bridge is designed using a force-based approach. Recent studies (Priestley et al., 2007) show that the force-based design may not necessarily guarantee the required target performances. The codes are now moving towards a performance-based design approach, which consider the design as per the target performances at the design stage. As the present study focus on the application of displacement-based approaches to pier design, a brief introduction of the two methods, force-based and displacement-based design is summarized in the following sections.
In case of pier consisting of two or more columns, the horizontal forces at the bearing be distributed on columns as required by appropriate analysis. If the piers consist of either multiple piles or trestle columns spaced closer than three times the width of piles/columns across the direction of flow, the group shall be treated as a solid pier of the same overall width and the value of K taken as 1.25 for working out pressure due to water current according to relevant Clause 213.7 of IRC:6. If such piles/columns are braced then the group should be considered a solid pier irrespective of the spacing of the columns.
Hollow piers shall be provided with suitably located weep-holes of 100 mm diameter for enabling

free flow of water to equalize the water levels on inside and outside; considering rate of rise/fall of flood/tide water. The pier walls should be checked for expected differential water-head/wave pressure and silt pressure. In absence of detailed calculations, a minimum difference of 1.5 m in water levels on two sides shall be assumed.

## FORCE BASED DESIGN METHOD:

Force Based Design Method (FBD) is the conventional method to design the metro bridge pier. In Force based design method, the fundamental time period of the structure is estimated from member elastic stiff nesses, which is estimated based on the assumed geometry of the section. The appropriate force reduction factor (R) corresponding to the assessed ductility capacity of the structural system and material is selected in the force-based design and applied to the base shear of the structure.

The design of a pier by force based seismic design method is carried out as per IS 1893: 2002 Code. The design procedure to find the base shear of the pier by FBD method is summarized below.

Step 1: The structural geometry of the pier is assumed.
Step 2: Member elastic stiffness are estimated based on member size.
Step 3: The fundamental period is calculated by:

$T = 0.075\ h0.75$

Where h = Height of Building, in m

Step 4: Seismic Weight of the building (W) is estimated.
Step 5: The design horizontal seismic coefficient Ah for a structure determined by

$Ah = ZISa/\ 2Rg$

Where, Z = Zone factor
I = Importance factor
R = Response reduction factor,
Sa/g = Average response acceleration coefficient
Z, I, R and Sa/g are calculated as per IS 1893:2002 Code.

Step 6: The total design lateral force or design seismic base shear force (VB) along any principal direction is given by

$VB = Ah\ W$

Where Ah = Design Horizontal Seismic Coefficient and
W = Seismic Weight of the Building

## DIRECT DISPLACEMENT BASED DESIGN METHOD

The direct displacement based seismic design (DDBD) is proposed by Priestley et al. (2007) is used in the present study to design a metro bridge pier. The design philosophy of DDBD is based on the determination of the optimum structural strength to achieve a given performance limit state, related to a defined level of damage, under a specified level of seismic intensity., Priestley et al. (2007). The pier designed by DDBD method gives the uniform risk factor for the whole structure.

The design procedure to find the base shear of the pier by DDBD method is summarized below.

Step 1: Yield Curvature is calculated by

$$\Phi Y = (2.10 * \varepsilon y\ )/hc$$

Where, εy is the yield strain and hc is the section depth of rectangular column

Step 2: Yield Displacement is calculated by

$$\Delta y= \Phi y\ (H + Lsp)2\ /\ 3$$

Where, H is the Column Height and Lsp is the Effective additional height representing strain penetration effects

Step 3: Design Displacement is lesser of

$$\Delta d = \theta d * H \text{ or } \mu * \Delta y$$

The ductility at design displacement is, $\mu = \Delta d / \Delta y$

Where, $\theta d$ = Drift limit

Step 4: Equivalent viscous damping

$$\xi eq = 0.05 + 0.444(\mu -1/ \mu \pi)$$

Step 5: Maximum spectral displacement is calculated from Design Displacement Spectra Given in Priestley et al. (2007).

Step 6: Design Strength/Base Shear is given by

$$VB = Ke \Delta d$$

Where, Ke = Effective Stiffness at peak response

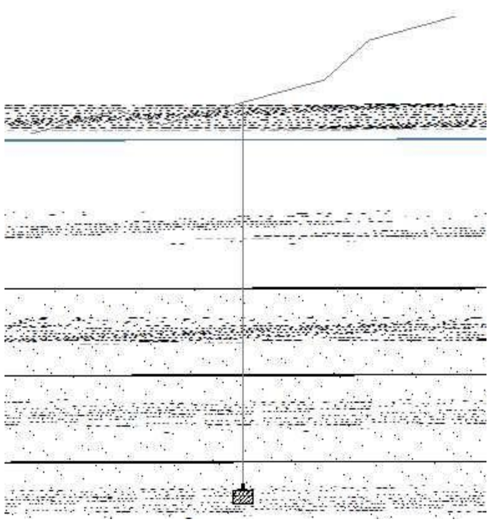
Te = Effective response period of pier

= Damping

c,n = Displacement at the corner period for n % damping

**DESIGN OF IER USING FORCE BASED DESIGN**

The geometry of pier considered for the resent study is based on the design basis report of the Bangalore Metro Rail Corporation (BMRC) Limited. The piers considered for the analysis are located in the elevated metro station structure. The effective height of the considered piers is 13.8 m. The piers are located in Seismic Zone II, as Per IS 1893 (Part 1): 2002. The modeling and seismic analysis is carried out using the finite element software STAAD ro. The typical pier models considered for the resent study are shown in figure



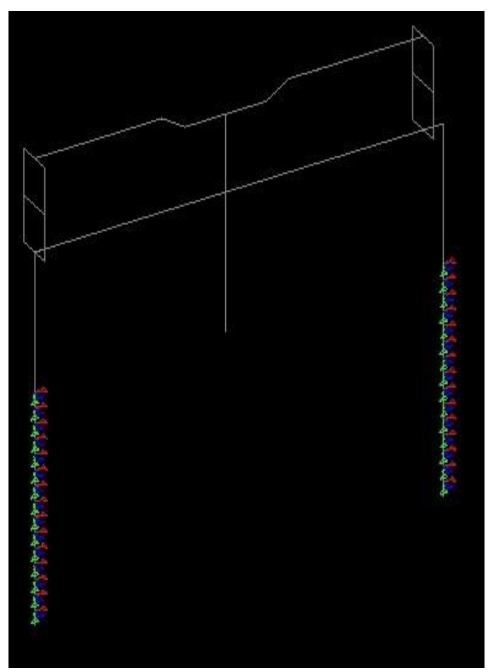


Figure : Typical pier Model

## MATERIAL PROPERTY:

The material property considered for the resent pier analysis for concrete and reinforcement steel is given in Table 1.

| | |
|---|---|
| **Properties of Concrete** | |
| Compressive Strength of Concrete | 60 N/mm$^2$ |
| Density of Reinforced Concrete | 24 kN/m$^3$ |
| Elastic Modulus of Concrete | 36000 N/mm$^2$ |
| Poisson's Ratio | 0.15 |
| Thermal Expansion Coefficient | 1.17 x 10$^{-5}$ /$^0$C |
| **Properties of Reinforcing Steel** | |
| Yield Strength of Steel | 500 N/mm$^2$ |
| Young's Modulus of Steel | 205,000 N/mm$^2$ |
| Density of Steel | 78.5 kN/m$^3$ |
| Poisson's Ratio | 0.30 |
| Thermal Expansion Coefficient | 1.2 x 10$^{-5}$ /$^0$C |

Table 1: Material property for pier

## Design Load

The elementary design load considered for the analysis are Dead Loads (DL), Super Imposed Loads (SIDL), Imposed Loads (LL), Earthquake Loads (EQ), Wind Loads (WL), Derailment Load (DRL), Construction & Erection Loads (EL), Temperature Loads (OT) and Surcharge Loads (Traffic, building etc.) (SR). The approximate loads considered for the analysis are shown in Table 2. The total seismic weight of the pier is 17862 kN.

The force-based design is carried out for pier as per IS 1893:2002 and IRS CBC 1997 Code and the results are shown in Table. From the FBD, it is found out that the minimum required cross section of the pier is only 1.5 m x 0.7 m for 2 % reinforcement. The base shear of the pier is 891 kN.

## DESIGN OF IER USING DIRECT DISPLACEMENT BASED DESIGN

The direct displacement based seismic design method proposed by Priestley et al. (2007) and IRS CBC 1997 Code is used to design of pier Type B and the results are shown in Table . The performance level considered for the study is a Life Safety (LS) level.

| Displacement Ductility | Drift Limit (m) | Cross Section (m) | Base Shear $V_b$ (kN) | Diameter of Bar (mm) | Number of Bars | % of Reinforcement Required |
|---|---|---|---|---|---|---|
| 1 | 0.276 | 1.5 x 0.7 | 604 | 32 | #16 | 1.2 % |
| 2 | 0.276 | 1.5 x 0.7 | 150 | 32 | #12 | 0.8 % |
| 3 | 0.276 | 1.5 x 0.7 | 86 | 32 | #12 | 0.8 % |
| 4 | 0.276 | 1.5 x 0.7 | 60 | 32 | #12 | 0.8 % |

The parametric study is carried to know the effect of displacement ductility on base shear for different performance levels and the results are shown in Figure 3.2. The figure shows that as the displacement ductility level increases the base shear of the pier decreases and also the difference between different performance levels is about 40 %.

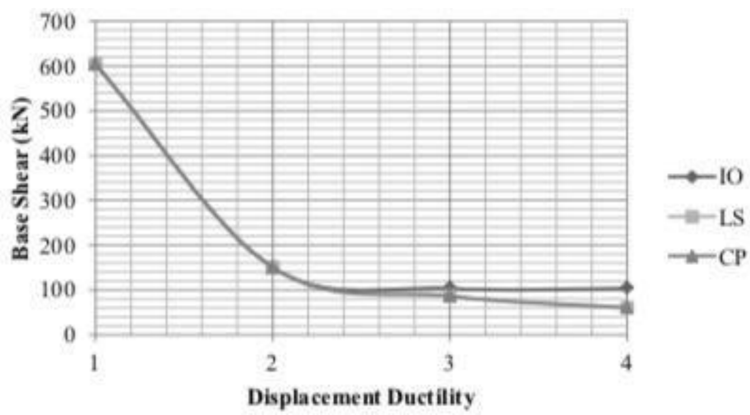



Fig Effect of displacement ductility on base shear for different performance levels

**PERFORMANCE ASSESSMENT**

The performance assessment is finished to check the performance of designed pier by Force Based Design Methodology and Direct Displacement Based Design Method. For this purpose,

Non-linear static analysis is conducted for the designed pier using Seismo Struct Software and the results are shown in Table. The section thought of is 1.5 m x 0.7 m. performance parameters behavior factor (R´), structure ductility (μ') and maximum structural drift (Δ' max) area unit found for each case.

The behavior factor (R´) is the ratio of the strength needed to keep the structure elastic. To the inelastic design strength of the structure. The behavior factor, R´, therefore accounts for the inherent ductility, over the strength of a structure and difference in the level of stresses thought of in its design. FEMA 273 (1997), IBC (2003) suggests the R factor in force-based Seismic design procedures. It's typically expressed within the following types of taking into account

The above 3 components,

$R = R\mu \times Rs \times Y$

Where, Rµ is the ductility dependent component also known as the ductility reduction issue, RS is the over-strength factor and Y is termed the allowable stress factor. With reference to Figure 3.3, in which the actual force–displacement response curve is perfect by a bilinear Elastic–perfectly plastic response curve, the behavior issue parameters may be defined as where, Ve, Vy, Vs and Vw correspond to the structure's elastic response strength, the idealized Yield strength, the primary important yield strength and the allowable stress design strength. The structure ductility, µ ', is defined in as maximum structural drift (Δ ´max) and the

Displacement corresponding to the perfect yield strength (Δ y)

In Force Based Design, a force reduction factor (R) of 2.5 is used, and the design base shear Is estimated to be 891kN in the FBD. The performance parameters of the section designed Using FBD shows that the be heavier factor R is found to be about 2.74. The same pier is designed using a DDBD method for target displacement ductility and drift, the performance parameters structural ductility and structural drift are found out for these cases. It shows that the achieved performance parameters are higher than assumed in the design stage in both Cases of DDBD. Though the FBD may not always guarantee the performance parameter required, in the resent case the pier achieves the target demand. In the case of DDBD,

The design considers the target displacement ductility and drift at the design stage, and the resent study shows that in each the examples the DDBD method achieves the behavior factors more than targeted Values. These conclusions can be considered only for the selected pier. For General conclusions large number of case studies is needed and it is treated as a scope of future work

| Designed | | | Type of design | $V_b$ (kN) | % of Steel | Φ (mm) | No. of Bars | Performance Parameters Achieved | | |
|---|---|---|---|---|---|---|---|---|---|---|
| µ | Δ | R | | | | | | µ | Δ | R |
| | | 2.5 | FBD | 891 | 2 % | 32 | #28 | | | 2.74 |
| 1 | 0.276 | | DBD | 604 | 1.2 % | 32 | #16 | 3.5 | 0.35 | 3.25 |
| 2 | 0.276 | | DBD | 150 | 0.8 % | 32 | #12 | 3.4 | 0.34 | 11.63 |

Performance Assessment of designed pier

## PARAMETRIC STUDY ON BEHAVIOUR OF CURVED BOX GIRDER BRIDGES

Parametric study of box girder bridges using finite element method is described in this paper. The parameters of box girder bridges considered in this study are radius of curvature, span length, span length to the radius of curvature ratio and number of boxes. The various responses parameters considered are the longitudinal stress at the top and bottom, shear, torsion, moment, deflection and fundamental frequency.

Numerical analysis carried out by Gupta et al. (2010) is used for validation of the finite element model. The parametric study is carried out, using 60 bridge models, to investigate the behavior of box girder bridges. Also, the results obtained from parametric study are discussed briefly in this paper.

## VALIDATION OF THE FINITE ELEMENT MODEL:

To validate the finite element model of box girder bridges in SAP 2000, a numerical example from the literature (Gupta et al., 2010) is considered. Figure 4.1 shows the cross section of simply supported Box Girder Bridge considered for validation of finite element model. Box girder considered is subjected to two concentrated loads (P = 2 X 800 N) at the two webs of mid span. Span Length assumed in this study is 800 mm and the material property considered are Modulus of elasticity (E) =2. 842GPa and Modulus of rigidity (G) =1. 015GPa.

The mid span deflection of the modeled box girder bridge is compared with the literature and it is resented in the Table 4.1. From the Table 4.1, it can be concluded that the resent model gives the accurate result.

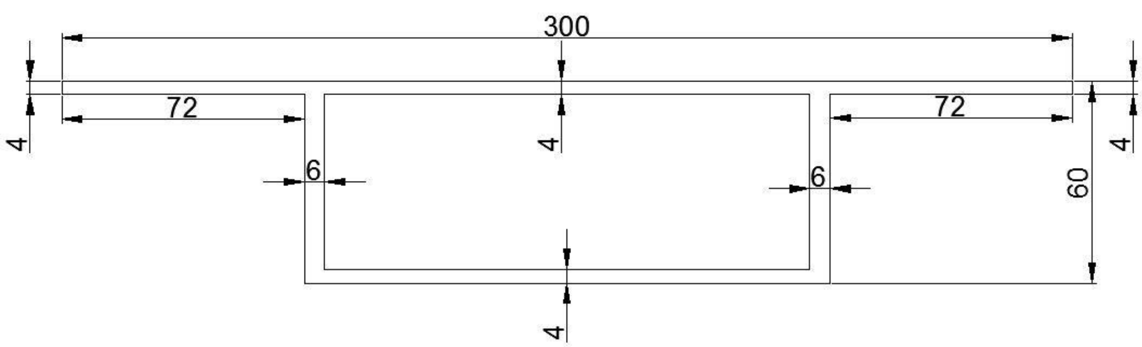
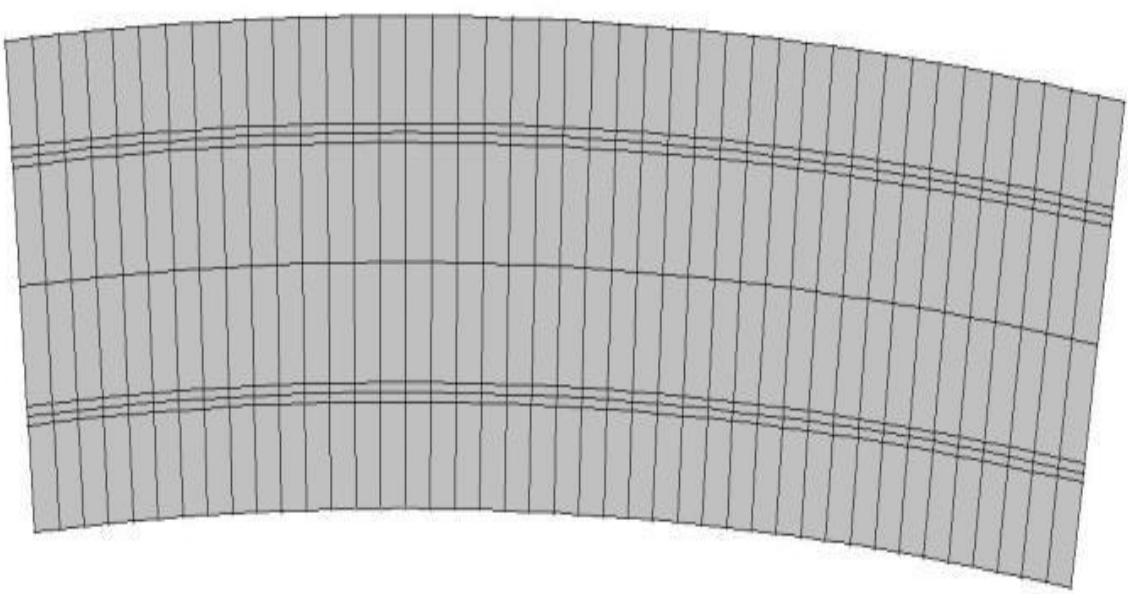



Figure 4.1: Cross Section of Simply Supported Box Girder Bridge

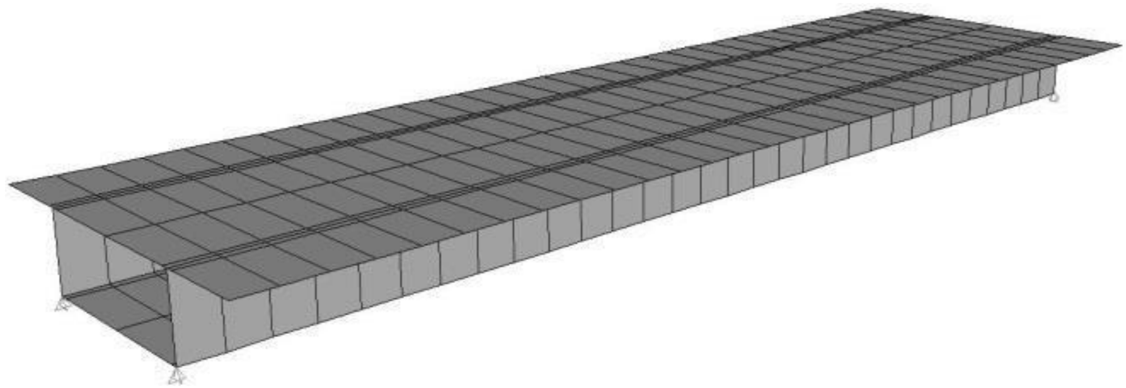
## FINITE ELEMENT MODELLING

The finite element modeling methodology adopted for validation study is used for the resent study. The modeling of Box Girder Bridge is carried out using Bridge Module in SAP 2000. The Shell element is used in this finite element model to discretize the bridge cross section. At each node it has six degrees of freedom: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z axes. The typical finite element discretize model of straight and curved simply supported box Girder Bridge in SAP 2000 is shown in figure 4.3(a) and 4.3(b).

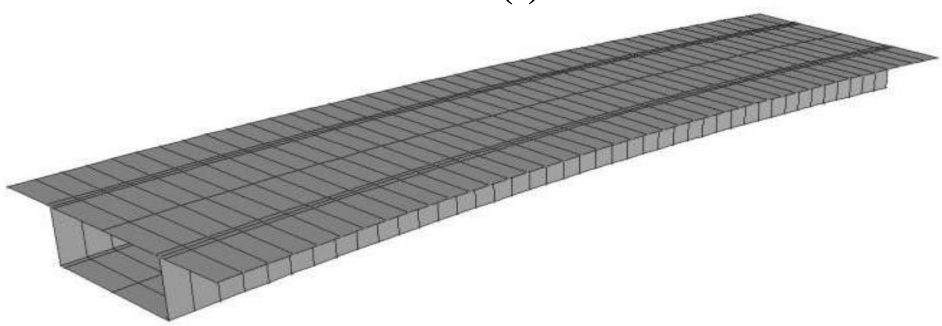
4.3(a) plan



4.3(b) plan



4.3(a) 3D model



4.3(b) 3D model

**PARAMETRIC STUDY**

The parametric study is carried out to investigate the behavior (i.e., the longitudinal stress at the top and bottom, shear, torsion, moment, deflection and fundamental frequency) of box girder bridges for different parameters viz. radius of curvature, span length, span length to radius of curvature ratio and number of boxes. In this paper, parametric study on behavior of box girder bridges is carried out by using finite element method. The numerical analysis of finite element model is validated with model of Gupta et al. (2010). The parameter considered in this paper to resent the behavior of SCBG, DCBG and TCBG bridges are radius of curvature, span length and span length to the radius of curvature ratio. Theses parameters are used to evaluate the response parameter of box girder bridges namely longitudinal stresses at the top and bottom, shear, torsion, moment, deflection and fundamental frequency of three types of box girder bridges. The results obtained from this parametric study are resented below:

From the parametric study it is found out that as the radius of curvature increases, responses parameter longitudinal stresses at top and bottom, shear, torsion, moment and deflection are decreases for three types of box girder bridges and it shows not much variation for fundamental frequency of three types of box girder bridges due to the constant span length.

It is observed that as the span length increases, responses parameter longitudinal stresses at the top and bottom, shear, torsion, moment and deflection are increases for three types of box girder bridges and fundamental frequency decreases for three types of box girder bridges.

It is noted that as the span length to the radius of curvature ratio increases responses parameter longitudinal stresses at top and bottom, shear, torsion, moment and deflection are increases for three types of box girder bridges and as span length to the radius of curvature ratio increases fundamental frequency decreases for three types of box girder bridges.

## SUMMARY AND CONCLUSIONS

### SUMMARY

A metro system is an electric passenger railway transport system in an urban area with a high capacity, frequency and the grade separation from other traffic. An elevated metro system is the most referred form of metro structure due to ease of construction and less cost compared to other types of metro structures. An elevated metro system has two major components pier and box girder. In this project, study has been carried out on these two major elements.

In the first art of this study, the performance assessment on designed pier by Force Based Design and Direct Displacement Based Design is carried out. The design of the pier is done by both force- based design method and direct displacement-based design method.

In the second art, parametric study on behavior of box girder bridges is carried out by using finite element method. The numerical analysis of finite element model is validated with model of Gupta et al. (2010). The parameter considered to resent the behavior of Single Cell Box Girder, Double Cell Box Girder and Triple Cell Box Girder bridges are radius of curvature, span length and span length to the radius of curvature ratio. These parameters are used to evaluate the response parameter of box girder bridges namely longitudinal stresses at the top and bottom, shear, torsion, moment, deflection and fundamental frequency of three types of box girder bridges.

### CONCLUSIONS

- The performance assessment of selected designed pier showed that,
- Force Based Design Method may not always guarantee the performance parameter required and in the resent case the pier just achieved the target required.
- In case of Direct Displacement Based Design Method, selected pier achieved the behavior factors more than targeted Values.
- These conclusions can be considered only for the selected pier. For General conclusions large numbers of case studies are required and it is treated as a scope of future work.
- The parametric study on behavior of box girder bridges showed that,
- As the radius of curvature increases, responses parameter longitudinal stresses at the top and bottom, shear, torsion, moment and deflection are decreases for three types of box girder bridges and it shows not much variation for fundamental frequency of three types of box girder bridges due to the constant span length.
- As the span length increases, responses parameter longitudinal stresses at the top and bottom, shear, torsion, moment and deflection are increases for three types of box girder bridges and fundamental frequency decreases for three types of box girder bridges.
- As the span length to the radius of curvature ratio increases responses parameter longitudinal stresses at the top and bottom, shear, torsion, moment and deflection are increases for three types of box girder bridges and as span length to the radius of curvature ratio increases fundamental frequency decreases for three types of box girder bridges.

- Analysis and design of the elevated Metro Bridge as per IRC codes (here IRC 70R loading) can be easily done by STAAD.Pro. In connection with STAAD.beava. Mechanism is well understood.
- The maximum resultant nodal displacement is for node 1529; 0...015mm in x, -51.203mm in y and -.287mm in x.
- The maximum resultant beam end displacement is for beam 1930 and node 1529 equivalent to 51.204.
- The maximum and minimum values for beam maximum forces by section property are computed for axial, shear and bending.
- The effect of vertical loading for 6 traffic lanes showing width, front clearance, and rear clearance, no. of axles, position in x, and position in y with orientation can be determined. The orientation varies from 0 to 1.5708.
- The concrete design for element 61 gives the top and bottom longitudinal reinforcement is 0.540 and 0.545. The top and bottom transverse reinforcement are 0.540 and 0.780 for element 61. Similarly, for another element, it can be found out.
- It is must for today's engineers, designers, research scholars to make an effective contribution to what is the purpose of each high-quality design and for the improvement of quality of environment in which we all are residing. Thus, evolution of software must be properly used so that it meets the beneficiary needs.

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